# Financial Econometrics

## Stellenbosch University

Session 1: Introduction

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Semester II

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# Introduction

- In this session, we briefly cover some of the most important portfolio risk measures.
- We will then in the practical fit these for a portfolio of assets to get a feel for what is typically used as risk and performance measures.
- We will make use especially of the package: PerformanceAnalytics, which we have seen in the practicals.
- We will also do some of the calculations by hand to get an idea of how it is done.

# References

- The notes and code in R were created using many references. I suggest working along mostly with Tsay (2012)'s chapter 7 and also Bacon (2011)'s textbook.
- For an application to SA data and comparison of various volatility models' ability to correctly predict VaR and ES, see this paper by Katzke and Garbers (2015).

# Goal

- The goal of risk measurements is to provide investors with a means to aid investment decision making.
- Markovitzian financial theory suggests that investors consider an asset's ability to deviate from expected returns as an indication of the risk of holding such an asset.
- Since the 1950s when financial theory emerged, investors have desired a **number** or quantitative measure which can be used to approximate an asset's risk.

# Broad measures of risk

- Volatility (or $\sigma$): how much an asset is expected to deviate from its mean.
- Value-at-Risk (VaR)
- Conditional VaR, or Expected Shortfall (ES)

# Volatility

Standard volatility measures typically consider $\sigma$, which is symetric around zero:

$$\sigma^2 = \frac{(r_i - \bar{r})^2}{n} \tag{1}$$

The problem with this measure is that it treats positive and negative returns equally. Despite investors fearing more the downside risk than the upside (assuming they are invested long, which most are).

$\sigma$ is appropriate only if returns are approximately normally distributed around the mean.

Sigma as a measure of risk remains widely used, as e.g. in the Sharpe Ratio:

$$Sharpe = \frac{R_p - R_{RiskFree}}{\sigma_P} \tag{2}$$

# Risk

There are several corrections made to the dispersion measure ($\sigma$) to make it more applicable to investment analysis.

Mean Absolute Deviation (MAD):

$$MAD = \sum_{i=1}^{n} \frac{|r_i - \bar{r}|}{n} \tag{3}$$

$d$-ratio: comparing total value of downside versus upside returns:

$$d = \frac{n_{-} \sum_{t=1}^{n} min(r_i, 0)}{n_{+} \sum_{t=1}^{n} max(r_i, 0)} \tag{4}$$

The lower the $d$-ratio the better.

# Risk

Downside Risk (similar to semi-deviation): focussing only on returns below Minimum Acceptable Return (MAR):

$$\sigma_{dd} = \sqrt{\sum_{i=1}^{n} \frac{R_i - {R_{MAR}}^2}{n}} \tag{5}$$

- Despite the existence of far more efficient measures of risk, most practitioners tend to use the simple $\sigma$ as the core measure of risk, which is appealing for its simplicity and that it is an unbiased proxy for risk.

# Value-at-Risk

- Measures such as VaR had been criticized after the recent GFC as a major contributing factor to the near global financial meltdown.
- This follows as much blame was placed on Financial Engineers that used such simplified measures (dependent often on normality assumptions), and notoriously not emphasizing the shortcomings of these measures.
    - The question, though, is whether the **models** should be blamed, the **FE** that designed and used the measures, or the **investors** who demanded simple measures for an immeasurable thing and using it without proper due diligence,
- But I will leave the phlosiphical arguments for another day. Today we discuss calculating VaR's and Conditional VaR's (ES), as it is still widely used as a measure of downside potential.

# Value-at-Risk: Definition

- VaR is defined as a single estimate of the amount by which an institution / investor's position in a risk category could decline because of general market movements during a given holding period Tsay (2012,:329).
- Note: events that fall outside of the broad term *general*, will not be picked up by the VaR estimate.
- Black Swans do not appear in past data, and thus will likely not feature in predicted risk estimates such as VaR.
- In our definition, we define the value of asset k at time t: $V_t$ and the Loss random variable as: $L_t(l) = V_{t+k} - V_t$.

# Value-at-Risk: Definition

The CDF of $L_t(l)$ is $F_l(X)$.

Thus, we assess big losses based on typically small probabilities (e.g. the 5% or 1% probabilities), such that the what is measured is tail events (rare losses)

Now, with probability $p$, the potential loss encountered by the holder of the financial position from $t$ to $t + l$ is $\leq VaR_{1-p}$, with:

$$VaR_{1-p} = inf \{x|F_l(X) \geq 1 - p\} \tag{6}$$

Implying:

$$Pr \{L_t(l) > VaR_{1-p}\} \leq p \tag{7}$$

VaR can be translated then as the probability that an asset holder would encounter a loss exceeding $VaR_{1-p}$ $\forall t \in [t, t + l]$.

# Value-at-Risk: Definition

- Note that we can rewrite equation 6 in terms of its quantile assuming a univariate CDF:

$$x_q = inf\left\{x|F_l(X) \geq q\right\}, \quad \forall q \in (0,1) \tag{8}$$

- This implies that VaR is simply the $(1-p)^{th}$ quantile of the loss distribution.
- You will see this to be the case in the practical when we do VaR by hand.

# Value-at-Risk: Definition

- Formally, assuming that returns are distributed $\mathcal{N}(0,1)$ the Value-at-Risk for both long and short positions are the values, $VaR^{(1-p)}$, such that:

$$p_{Long} = P\left(x_t \leq VaR_t^{(1-p)}\right) = \int_{-\infty}^{VaR_t^{(1-p)}} \frac{1}{\sqrt{2\pi}} exp\left(-\frac{1}{2}\, x_t^2\right)\, dx_t \quad (9)$$

$$p_{Short} = P\left(x_t \geq VaR_t^{(1-p)}\right) = \int_{VaR_t{}^{(1-p)}}^{+\infty} \frac{1}{\sqrt{2\pi}} exp\left(-\frac{1}{2}\, x_t^2\right)\, dx_t \quad (10)$$

# Value-at-Risk: Definition

- Equations 9 and 10 then simply imply that:

$$VaR_t^{(1-p)} = \zeta_p \tag{11}$$

Where $\zeta_p$ is the $100p^{th}$ percentile of the standard normal distribution. A more generalized notation uses the case where $x_t \sim \mathcal{N}(\mu_t, \sigma_t^2)$. In this case we have (for the long position):

$$p = P\left(x_t \leq VaR_t^{(1-p)}\right) = \frac{1}{\sigma_t\sqrt{2\pi}} \int_{-\infty}^{VaR_t^{(1-p)}} exp\left(-\frac{1}{2}\left(\frac{x_t - \mu_t}{\sigma_t}\right)^2\right) dx_t. \tag{12}$$

From which we get the VaR as:

# Value-at-Risk: in R

- From our definitions, it follows simply that VaR is the $q^t h$ quantile - denoting the area under the CDF to the left (long) or right (short) of a point on the probability axis. Suppose we have normal returns, $x_t$, with first two moments: $\mu = 20$ & $\sigma = 8$. (See function qnorm).

mu <-20 sigma <- 8 alpha <-0.95 # To get the 95% VaR estimate: var = qnorm(alpha, mu, sigma) print(var)

# Value-at-Risk: in R

If we again use the return series $X_t$, we calculate VaR using normal distribution:

mu <-20 sigma <- 8 alpha <-0.95 # To get the 95% VaR estimate: var = mu+sigma*qnorm(alpha, 0, 1) print(var)

And using a t-distribution (assuming $X_t$ has d.o.f.: $n - 1 = 900$), as:

dof = 20 var.t = mu+sigma*qt(alpha, dof) print(var.t)

Note in this case they are rather similar, as we know $t$-distr $\sim \mathcal{N}$ as $n \to \infty$

# Expected Shortfall (ES)

- Now we discuss a more informative version of VaR, the Conditional VaR, or Expected Shortfall (ES).
- Simply put, ES is the expected loss of a financial position after a catastrophic event.
- It thus takes into account the expected loss of $X_t$, **given that** $X$ exceeds its VaR. Thus it measures both the probability and cost of extreme events.

$$ES_{t+1|t}^{(1-p)} = E\left[x_{t+1}| \left(x_{t+1} \leq VaR_{t+1|t}^{(1-p)}\right)\right] \tag{14}$$

$$ES_P = \frac{\int_{VaR_p}^{\infty} l \times f_L(l)dl}{p} \tag{15}$$

# ES: Definition

- If normality applies, the difference between the VaR and the ES estimate can be given as $VaR_t^{(0.95)} = -1.645$ and $ES_t^{(0.95)} = -2.061$, respectively.
- Thus at a 95% level of confidence, a portfolio of size $100 000 is predicted to lose more than $1645 5 days out of a hundred, whilst the average daily loss given that a VaR violation occurs, is estimated to be $2061.
- If $u = F(X)$ for $VaR \leq x \leq \infty$, then ES can be seen to average all $VaR_u$ for $(1 - p) \leq u \leq 1$

# ES in R:

For normal data with: $\phi(z) = f_z(z)$, the pdf of $Z \sim \mathcal{N}(0,1)$

$$ES_\alpha = \mu_l + \sigma_l \times \frac{\phi(q_\alpha^z)}{1-\alpha} \tag{16}$$

mu <-20 sigma <- 8 alpha <-0.95 # To get the 95% ES estimate: phi.qaz <-qnorm(alpha) # This is the phi(q_alpha^z) above ES = mu+sigma*(dnorm(phi.qaz)/(1-alpha))

print(ES)

# Cornish Fischer modifications

- Cornish-Fisher suggested modifying VaR and ES estimates by taking into account skewness and kurtosis, which typically deviate from standard distributional assumptions for returns series (see this technical paper for a deeper discussion). The literature, in fact, suggests that the modified version should be preferred.
- Building from equation 13, we define the CF-VaR, or MVaR as:

$$
\begin{aligned}
VaR_t^{(1-p)} &= \mu_t(X) + \sigma(X).z_{CF} \\
z_{CF} &= q_p + \frac{(q_p^2 - 1)S(X)}{6} + \frac{(q_p^3 - 3q_p)K(X)}{24} - \frac{(2q_p^3 - 5q_p)S^2(X)}{36}
\end{aligned} \tag{17}
$$

- The modified ES is similarly adjusted for taking into account the 3rd and 4th moments.

# Summary

- Historical method:
- Simplest method: it basically uses historical data to estiamte the 5% and 1% worst (extreme) outcomes - implying the loss wil not exceed this estimate at 95% and 99%, respectively. This approach is exposed to historical bias.
- Monte-Carlo simulation
- Generate a large number of samples from a distribution that mimicks the historical data approach, and then again estimate the extreme 5% and 1% levels.
- Parametric
- Parametric estimates assume an underlying distribution that mimicks the

# Off to the practical

- We will now fit these and other risk measures in R in the practical session.
- We will also discuss how to automatically fit these measures using packages, but it helps to see how these are done by hand too...

# References

Bacon, Carl R. 2011. *Practical Portfolio Performance Measurement and Attribution*. Vol. 568. John Wiley & Sons.

Katzke, Nico, and Chris Garbers. 2015. "Do Long Memory and Asymmetries Matter When Assessing Downside Return Risk?" *Investment Analyst Journal*.

Tsay, Ruey S. 2012. *An Introduction to Analysis of Financial Data with R*. Wiley Publishing.