The Monkeys Prevail: Questioning the Efficiency of Cap-Weighted Indexes

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Abstract

The market capitalisation weighted index has traditionally been considered the standard benchmark for investment managers. This follows from the canonical theoretic foundations in finance, which argue for the efficiency of mimicking the composition of the market by investing proportional to size. Such capitalization (size)-weighted portfolios have gained prominence since, and have become a popular investment alternative to active (and expensive) funds. However, various studies have questioned the efficiency of such a capweighting approach. In this paper, we consider the performance of 10 000 random monthly reweighted portfolios against the benchmark cap-weighted and equal-weighted index on 16 major markets worldwide. This is done in order to guage the efficiency of cap-weighting relative to the uninformative (or random) weighting mechanism employed. Our results show that in 11 out of the 16 universes under investigation our random (or monkey) selected portfolio weights outperform the cap-weighted index on a risk adjusted basis; for the remaining 5 universes, we find the result could be explained by exceptional performance among large stocks over the sample period under consideration.

Keywords: Monkey Portfolio, Market Capitalisation Weighted Index, Equal Weighted Index

JEL classification

1. Introduction

The market capitalisation weighted (cap-weighted) index has been the standard benchmark for active investors and become a popular investment vehicle for passive investors. Often considered and marketed as providing investors with a fair return at low cost, the proof of its efficiency is often argued to be grounded in theory. This follows from foundational portfolio management theory emanating from the canonical work done by Markowitz, in which a representable and

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investable market portfolio is suggested as an efficient alternative to holding an actively selected portfolio of stocks. Such portfolio constituent weights are then calculated proportional to the size of the stocks relative to the index. Arguing, then, for the efficiency of such portfolios implies implicitly arguing for the effectiveness of size as a signal of future return potential. This, however, has consistently been shown not to be the case in academic literature (c.f, among many papers, Haugen and Baker (1991) & J. Hsu (2004)). While efficiency gains are realized from the lowering of costs due to the simplicitly of the weighting scheme¹, several studies have argued for its implicit portfolio construction inefficiencies.²

In this paper we extend the work done by Clare, Motson, and Thomas (2013) in comparing the performance of cap-weighted indexes to a large set of randomly weighted (monkey) portfolios. As initially argued by Malkiel (1999), if such randomly constructed portfolios, void of any useful information, manage to outperform the cap-weighted portfolio, it should suggest clear inefficiencies to this popular approach of portfolio weighting. Our paper extends that of earlier research by studying various stock exchanges across the development spectrum and with widely differing levels of concentrations.³ Our prior hypothesis is that if cap-weighted portfolios provide fair and efficient returns, it should at least perform in the middle range of the randomly weighted portfolios. Our findings, echoing that of Clare, Motson, and Thomas (2013) & Malkiel (1999), suggest otherwise and highlight the performance inefficiencies of popular cap-weighted indexes.

In what follows we compare the performance of 10 000 randomly generated monkey portfolios against the benchmark cap-weighted and equal-weighted index of 16 stock markets worldwide of varying size, level of economic development and concentration. We then compare the risk adjusted performance of our monkey portfolios against the cap-weighted alternatives in terms of standard

¹This follows as managers simply invest proportional to relative size, thus not incurring research or significant operational costs, while also limiting turnover.

 $^{^{2}}$ Examples of these inefficiencies include a ack of diversification in concentrated markets, as well as empirical studies showing forward return signals to be negatively correlated with size.

 $^{^{3}}$ Concentration in this context refers to a cap-weighted index having a high exposure to a small set of very large stocks.

measures of portfolio performance (we compare the Sharpe Ratios and Maximum Drawdowns experienced by the respective portfolios).

Our results show that in 11 out of the 16 universes under investigation the monkey portfolios exhibit far superior performance to that of the cap-weighted indexes. In fact, in cases where monkey portfolios generally are superior, it is not only on an absolute basis, but also on a risk adjusted basis. Also, in the few cases where cap-weighted portfolios manage to outperform the monkeys on a consistent basis, the results can be explained by large cap stocks delivering consistently superior results to that of the other stocks collectively. We believe that those findings should be regarded as a function of its past, and not regarded as indicative of the more general claim of cap-weighting efficiency more generally.⁴

In addition to the above mentioned result, we also consider the effect of adding restrictions to the weights of the random portfolios. Specifically, we constrain the random weights to be more closely distributed to the equal weighted index in order to consider the effect of a less concentrated monkey portfolio. The results, as expected, show distributions that are more closely distributed around the equal-weighted index, which in nearly all cases produce improved results relative to the respective cap-weighted indexes.

The rest of the paper is set out as follows. In section 2 a literature review is conducted considering the case for and against the efficiency of cap-weighted indices as well as similar random portfolio studies conducted. Section 3 describes the data used and section 4 discusses our methodology. Section 5 presents the results along with a discussion of our findings. Section 6 concludes.

⁴This follows as there is certainly no empirical or theoretic basis for arguing that large cap stocks should continue producing superior returns in such cases.

2. Literature Review

In his book, A Random Walk down Wall Street, Malkiel (1999) makes the assertion that even a blindfolded monkey throwing darts to pick stocks would be able to outperform the average fund manager in the U.S. He backs up this statement with the claim that two thirds of stock managers fail to outperform the S&P 500 Stock Market Index. This is then argued to provide evidence that individual investors would likely in the future be better off simply buying and holding a (any) broad and diversified portfolio of stocks within the universe. The question, then, is whether such a passive strategy should be aligned to the size of stocks within a universe (cap-weighted), or according to the amount of stocks (equal weighting).

2.1. Is cap-weighting an efficient investment?

The cap-weighted index is an index composed of stocks weighted by their market capitalization, implying the largest stocks in the universe will have the biggest weighting in the index. In addition to producing clear benefits in terms of fees (due to its intuitively appealing simplicity in design), high liquidity (being overweight large stocks, the expectation is of improved liquidity), the most contentious benefit is in it being a mean-variance efficient portfolio (c.f. the critique levelled at this strategy discussed in J. Hsu (2004)).

The cap-weighted index is often considered theoretically consistent with the Capital Asset Pricing Model (CAPM) and as such tend to be considered mean-variance efficient if the market is efficient.⁵ However, as argued by Clare, Motson, and Thomas (2013, 4), there is no sound *ex-ante* reason to believe that the cap-weighted index would be mean-variance efficient. J. Hsu (2004, 4) argues further that cap-weighted indices, by virtue of construct, are sub-optimal due to the fact that it overweight expensive stocks, while underweighting stocks with prices that are low compared to their fundamentals. Perold (2007, 4) further argues that given the presupposition that if the

⁵Mean-variance efficiency in this context refers to a portfolio earning a fair return for the given aggregate market risk, where earning higher returns require accepting higher risk.

cap-weighted index can be outperformed without any knowledge of fair value, then even if prices move towards their fair value the direction of this movement would be random.

A large body of literature has emerged that suggest systematically identifiable factors can be used to construct portfolios that outperform cap-weighted indexes consistently. Amenc, Goltz, and Le Sourd (2009), e.g., considers whether indices based on firm characteristics, or fundamentals, generally outperforms cap-weighted indices in the U.S. They consider the performance of 9 different fundamental indices against the cap-weighted index over the period January 1998 to December 2006. It is found that although all indices outperforms the S&P 500, most of the out-performance are not statistically significant and most fail to outperform the equal weighted index. By contrast, Arnott, Hsu, and Moore (2005), among many others, find that the S&P 500 index under-performs fundamental weighted indices over a 42 year period ranging from 1962 to 2003 in the U.S. The authors further argue that their results are robust across time, business cycles, up and down markets an rising and falling interest rate regimes.

Due to the disagreement that exists in empirical studies regarding the performance of cap-weighted indices, an alternative way to evaluate the efficiency of such cap-weighted indices empirically is needed. Considering the performance of portfolios with random weights assigned to stocks in the benchmark index against the performance of the cap-weighted indices is a valid alternative technique. Should random portfolios be found to outperform the cap-weighted index on a risk adjusted basis, it would be strong evidence for the inefficiency thereof. This would, furthermore, provide the impetus for further studying the identification of informative (systematic) weighting strategies that could be utilized to outperform market returns.

2.2. Monkey Portfolio Studies

Clare, Motson, and Thomas (2013) conducted a study in which they compared alternative weighting schemes to that of the cap-weighted index using data on the largest 1000 US stocks between 1968 and 2011. The authors compare the distribution of 10 000 000 randomly generated portfolios to the cap-weighted index. Quite surprisingly, almost every monkey portfolio outperforms the capweighted index in absolute terms. If, as according to efficient market theorists, the cap-weighted index is mean-variance efficient, then the randomly reweighted portfolios should more risky (referring to our earlier definition, this is required to explain this outperformance). However, Clare, Motson, and Thomas (2013, 24) shows that when comparing the Sharpe ratios of the monkey portfolios to that of the cap-weighted index, most of them outperform the index also on a risk adjusted basis. This result shows strong evidence that, for the US, the cap-weighted index displays glaring inefficiencies over the specified sample period.

To the best of our knowledge no study comparing random portfolios to market indices across multiple stock universes comparable to this study exists. In this study we aim to fill this gap in the literature to make a more general conclusion about the expected efficiency of cap-weighted indices.

3. Data

The data used in our study encompasses all the stocks of several universes across the development spectrum. We avoid survivorship bias by reweighting each month's constituent information to a point-in-time estimate using Bloomberg's EQS functionality. We then create cap-weighted indexes for each universe by re-weighting monthly. For each company in our sample, we consider the daily net dividend reinvested total returns, adjusted for stock splits, rights issues, spin-offs, and other possible distortive pricing effects. We then calculate the daily returns for each stock in the respective universes.

In order to make our results practically applicable, we add constraints to some of our included universes. This includes, e.g., limiting the scope of stocks included to 95% of market cap in certain instances (this is done in order to limit exposure to illiquid and highly variable stock returns that would typically be avoided by active fund managers).

In an effort to make our results general and relevant, we consider universes of vastly different sizes and concentrations. Our inclusion of European stocks, e.g., include the popular DAX30 index constituents comprising of only 30 stocks (no cap-weight trimming), while the SPEURO consists of roughly on aggregate 154 stocks (taking the largest 95% of stocks). We also consider large and efficient markets such as the Taiwan Stock Exchange, the Nikkei, the UK top 100, Kospi, Singapore's Straight and the SP500. Our sample also consists of emerging market indexes that have high levels of concentration (including South Africa's JSE All Share Index capped at 95% of cumulative market cap, as well as its Top 40 index, as well as the stock exchanges of Turkey, India and Brazil), as well as commodity and energy markets such as Australia, New-Zealand and Canada.

Our sample generally spans from April 2003 to January 2017 (while some universes start later due to data and constituent availability) and consists of daily returns series. Table 3.1 summarises the data used in this study, as well as specifying the adjustments made to each stock universe. The amount of stocks shown for each universe is approximate in the case where an universe is filtered with regards to cumulative market cap.

	Universe	Country	Stocks	CumMarketCap	Sample Period
1	AUS	Australia	158	0.95	21-03-2003 to 30-11-2016
2	CAN	Canada	196	0.95	18-03-2003 to 30-11-2016
3	CNX	India	175	0.95	13-01-2012 to 30-11-2016
4	DAX30	Germany	30	1.00	18-03-2003 to 30-11-2016
5	IBX	Brazil	61	0.95	18-03-2003 to 30-11-2016
6	JALSHAll	South Africa	84	0.95	18-03-2002 to 30-11-2016
7	JSE40	South Africa	40	1.00	18-03-2002 to 30-11-2016
8	KOSPI	South Korea	134	0.95	18-03-2003 to 30-11-2016

Table 3.1: Index Description

9	NKY	Japan	164	0.95	18-03-2003 to 30-11-2016
10	NZX	New Zealand	53	0.95	18-03-2003 to 30-11-2016
11	SP500	United States	500	1.00	18-03-2003 to 30-11-2016
12	SPEURO	European Union	154	0.95	14-05-2003 to 30-11-2016
13	STRAIT	Singapore	136	0.95	17-03-2008 to 30-11-2016
14	TUR	Turkey	80	0.95	16-03-2004 to 30-11-2016
15	TWSE	Taiwan	374	0.95	18-03-2003 to 30-11-2016
16	UK	United Kingdom	100	1.00	01-01-2003 to 30-11-2016

4. Methodology

In this section we describe the process of generating 10 000 random monthly reweighted portfolios in order to compare the performance to the market-cap weighted, as well as equally weighted, indexes for each stock universe considered. Each randomly weighted portfolio is constructed as follows. First, 10 000 random numbers are generated for each month using an uniform random distribution. These random numbers are then used to construct random monthly portfolio weights for all stocks in the chosen universe. This implies that each monkey portfolio would be re-weighted at the beginning of each month, with stocks held in this proportion for the remainder of the month. The randomly weighted daily stock returns for each of the 10 000 monkey portfolios are then calculated. These 10 000 random portfolio return series are then used to generate a distribution of cumulative returns and Sharpe ratios to be compared to the benchmark portfolios. This process is then simulated on all 17 Stock Universes. The cap-weighted and equal-ewighted portfolios then comprise of the same universe of stocks as held by the monkey portfolios and are also reweighted on a monthly basis in order to ensure comparability. We expect to see the equal-weighted portfolio, by design, to fall roughly in the middle of our distribution of random portfolios, while the cap-weighted alternative could either be above or below the equal weighting. We also assume a long-only strategy for all our estimated portfolios, as well as assuming that we are 100% invested (no cash holdings, and weights sum to unity).

We omit a consideration of trading or management costs in our analysis. This follows for two reasons. First, as we are studying claims of the theoretic efficiency of a cap-weighted portfolio (within the traditional mean-variance context), we remain consistent with canonical theories in ignoring fees and costs, as well as constraining our investment process to long-only strategies. Secondly, we aim to make inferences regarding the signal efficiency (or return predicting ability) of cap-weighted indexes, and are not suggesting adopting any particular monkey strategy.⁶ For this reason, we feel adding trading costs (the accurate calculation of which itself is problematic) would not be consistent with the research question we set out to ask: are cap-weighted indexes informationally efficient?

5. Results

5.1. Unrestricted Portfolios

As a first step, we consider unrestricted portfolio weightings. This implies, monkeys could assign any arbitrary weight to a stock in the portfolio (maximum of 100% and minimum of 0%).

Figure 5.1 represents the results of the 10 000 monkey portfolios on 16 indices against the capweighted and equal-weighted portfolio respectively. The solid lines represent the terminal value of \$1 invested in the cap-weighted portfolio at the start of the sample period, while the dotted lines represent the terminal value for \$1 invested in the equal-weighted index. The results show, almost unambiguously, that most monkey portfolios outperform the cap-weighted index for most universes. The only exceptions are Australia, Canada, Taiwan, Turkey and Singapore. These exceptions are considered in more detail in the next subsection, where we look at whether these

⁶Albeit tempting in some of the universes studied, employing a monkey to manage a portfolio would ultimately be an unpalatable strategy, despite extremely low management fees.

performances could be explaind by consistently high returns from large caps. Table 5.1 summarises the average terminal value (CumRet) of each portfolio for each universe as well as the proportion of monkeys that outperforms the the cap-weighted portfolio.

The equal-weighted terminal value is almost in the middle of the monkey terminal value distribution in every case. This is expected given that the random weights for each stock in the universe would be distributed uniformly around the equal weighting for that universe. The equal weighted portfolio thus also mostly outperforms the cap-weighted portfolio (except in cases where large stocks consistently outperformed the rest).

In reality, the true measure of a success for an investment strategy is the risk-adjusted return. Figure 5.2 evaluates the Sharpe ratios of the 10 000 monkey indices against that of the cap-weighted and equal-weighted index. In the case of every portfolio where the monkeys outperform the cap-weighted portfolio, the Sharpe ratio of the monkeys also outperform the Sharpe ratio of the cap-weighted portfolio. Table 5.1 shows the average Sharpe ratio as well as the average maximum drawdowns for the monkey portfolios. Except for the JALSH, JSE 40, Nikkei and IBX, the cap-weighted portfolio shows a marginally superior performance to the average of the monkey portfolios with regards to maximum drawdowns.



Terminal Value Distributions of 10 000 Monkeys

Figure 5.1: Terminal Value Distributions. Solid line = cap-weighted index. Dotted line = equal-weighted index.



Sharpe Ratio Distributions of 10 000 Monkeys

Figure 5.2: Sharpe Ratio Distributions (Rf = 0). Solid line = cap-weighted index. Dotted line = equal-weighted index.

	Universe	Portfolio	CumRet	Maxdrawdown	Sharpe(Rf=0)
1	AUS	Cap	2.10	-0.59	2.44
2	AUS	EW	1.79	-0.68	2.12
3	AUS	Monkeys	1.75	-0.67	2.07
4	CAN	Cap	1.90	-0.54	2.25
5	CAN	EW	1.85	-0.58	2.24
6	CAN	Monkeys	1.81	-0.57	2.18
7	CNX	Cap	1.47	-0.23	3.73
8	CNX	EW	1.64	-0.28	4.31
9	CNX	Monkeys	1.62	-0.28	4.24
10	DAX30	Cap	2.16	-0.60	2.33
11	DAX30	EW	2.72	-0.62	2.85
12	DAX30	Monkeys	2.77	-0.61	2.90
13	IBX	Cap	3.25	-0.61	2.92
14	IBX	EW	3.76	-0.57	3.38
15	IBX	Monkeys	3.64	-0.58	3.32
16	JALSHAll	Cap	3.53	-0.54	3.24
17	JALSHAll	EW	6.18	-0.47	5.60
18	JALSHAll	Monkeys	6.04	-0.46	5.59
19	JSE40	Cap	3.65	-0.56	3.20
20	JSE40	EW	7.64	-0.45	5.15
21	JSE40	Monkeys	7.44	-0.45	5.11
22	KOSPI	Cap	1.82	-0.60	1.93
23	KOSPI	EW	2.35	-0.64	2.43
24	KOSPI	Monkeys	2.27	-0.64	2.37

 Table 5.1: Summary Statistics of Portfolios

25	NKY	Cap	1.10	-0.72	0.91
26	NKY	EW	1.35	-0.71	1.31
27	NKY	Monkeys	1.34	-0.70	1.29
28	NZX	Cap	2.46	-0.43	4.62
29	NZX	EW	3.04	-0.45	6.27
30	NZX	Monkeys	3.04	-0.45	6.06
31	SP500	Cap	1.93	-0.63	2.16
32	SP500	EW	2.35	-0.67	2.52
33	SP500	Monkeys	2.38	-0.66	2.57
34	SPEURO	Cap	1.40	-0.64	1.39
35	SPEURO	EW	1.70	-0.66	1.84
36	SPEURO	Monkeys	1.69	-0.65	1.83
37	STRAIT	Cap	1.00	-0.59	0.54
38	STRAIT	EW	0.83	-0.66	-0.30
39	STRAIT	Monkeys	0.82	-0.65	-0.43
40	TAIEX	Cap	1.56	-0.62	1.63
41	TAIEX	EW	1.16	-0.69	0.95
42	TAIEX	Monkeys	1.14	-0.68	0.92
43	TUR	Cap	2.29	-0.66	2.36
44	TUR	EW	2.15	-0.68	2.33
45	TUR	Monkeys	2.10	-0.68	2.29
46	UK	Cap	1.75	-0.52	1.91
47	UK	EW	2.25	-0.58	2.49
48	UK	Monkeys	2.27	-0.57	2.54

5.2. Exceptions

Table 5.2 shows the average proportion over the sample period of the total index weight that make up the top decile (top 10%) of stocks for the cap-weighted and equal-weighted index respectively.

	Universe	Cap-Weight Top 10%	Equal-Weight Top 10%
1	AUS	0.62	0.10
2	CAN	0.48	0.10
3	CNX	0.48	0.10
4	DAX30	0.25	0.10
5	IBX	0.52	0.10
6	JALSHAll	0.57	0.10
7	JSE40	0.47	0.10
8	KOSPI	0.49	0.10
9	NKY	0.40	0.10
10	NZX	0.40	0.10
11	SP500	0.48	0.10
12	SPEURO	0.33	0.10
13	STRAIT	0.58	0.10
14	TAIEX	0.64	0.10
15	TUR	0.46	0.10
16	UK	0.48	0.10

Table 5.2: Top Decile Proportion of Index Weight

All of the cap-weighted indices are highly concentrated compared to the equal-weighted index. Considering in particular the universes where the monkey portfolios fail to outperform the capweighted index, the top decile of stocks make up a large proportion of the cumulative weight, between 46% in the case of Turkey and up to 64% in the case of Taiwan. This serves to show that in all of the exceptions, the performance of the cap-weighted index is attributable largely to the top decile of stocks in that index. It is therefore an useful exercise to consider how the top decile performed against the rest of the stocks in the universes where the monkeys failed to outperform the cap-index. We find that in each case the result is driven by a small concentrated set of large stocks in the universe consistently outperforming the rest.



Cumulative Return of Top Decile Against the Rest

Figure 5.3: Cumulative Return of Top Decile Against the Rest.

Figure 5.3 shows the performance of the top decile of stocks in each universe relative to the rest

of the stocks in that universe. The stocks are equally weighted in order to ensure an accurate comparison in the returns between large cap and small cap stocks. In every universe, except for Turkey, the top decile performed very well compared to rest of the stocks in the universe.

What this serves to show is that the performance of these universes were not caused by the fact that the cap-weighted index is a superior weighting scheme, but rather by a few large stocks performing particularly well in these indices. Turkey is the only exception to this rule where midcaps performed well relative to smaller caps and large caps. If the market-cap index was truly an efficient investment approach, we should expect that all universes follow the pattern shown by Turkey.

5.3. Restricted Portfolios

In addition to the unrestricted random portfolio simulations, we consider the effect of adding restrictions to the weights of the random portfolios. More specifically, we constrain random weights to fall within a more narrow band around the equal weighted index according to the following specification:

$$UpperLimit = EW + EW/2 \tag{5.1}$$

$$LowerLimit = EW - EW/2 \tag{5.2}$$

Where EW is the equal weight assigned to each stock in a specific universe. These restrictions are added in order to evaluate the effect of less concentrated random portfolios on the performance of these portfolios against the performance of the cap-weighted index. Figure 5.4 shows the distribution of terminal values for the JSE 40 and DAX 30 Indices respectively. These two indices serve as examples to illustrate the general result.



Terminal Value Distributions of 10 000 Monkeys

Figure 5.4: Terminal Value Distributions. Solid line = cap-weighted index. Dotted line = equal-weighted index.

All that the results serves to show is that the more diversified a monkey portfolio is, the more that portfolio would approximate the performance of the equal weighted index. Compared to the graphs of the JSE 40 and the DAX 30 in figure 5.1, terminal values of the monkey portfolio is distributed tighter against the equal weighted index. Table 5.3 summarises the performance of the restricted monkey portfolios for these two indices. In the case of both the DAX 30 and the JSE 40 the average terminal values (CumRet) of the monkeys is closer to the terminal value of the equal-weighted than under the unrestricted results in Table 5.1.

 Table 5.3: Summary Statistics of Restricted Portfolios

	Universe	Portfolio	CumRet	Maxdrawdown	Sharpe(Rf=0)
1	DAX30	Cap	2.16	-0.60	2.33

2	DAX30	EW	2.72	-0.62	2.85
3	DAX30	Monkeys	2.72	-0.62	2.84
4	JSE40	Cap	3.65	-0.56	3.20
5	JSE40	EW	7.64	-0.45	5.15
6	JSE40	Monkeys	7.64	-0.46	5.13

6. Conclusion

In this paper we considered the performance of 10 000 randomly weighted portfolios (monkey portfolios) against the performance of the cap-weighted index on 16 stock universes. It was found that in 11 out of the 16 universes the monkey portfolios outperformed the cap-weighted index on a risk adjusted basis. In the case where the monkey portfolios failed to outperform the cap-weighted index it was shown that, with the exception of Turkey, this performance was driven by a few large stocks performing very well.

The implication of this finding is that the cap-weighted index is not an efficient investment. For the same expected return, even a random portfolio has a significantly lower volatility. From an investment perspective, the findings imply that a passive investor should at least prefer a simple equal-weighted index over a cap-weighted index, while the cap-weighted index might be an inferior benchmark for active investors.

This study can be extended by considering whether the results are consistent over different time periods and different re-weighting periods (e.g. quarterly, yearly). In addition, the performance of the random portfolios can be compared to that of the average fund manager performance in each stock universe respectively.

7. Appendix

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